Technical Notes

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Effective Thermal Property Improves Phase Change Paint Data

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Nomenclature

c = specific heat, J/kg K
h = aerodynamic heat-transfer coefficient, W/m^2 K
k = thermal conductivity, W/m K
$\dot{q}$ = radiative heat flux, W/m^2
T = temperature, K
T_{aw} = adiabatic wall temperature, K
t = time, sec
x = spatial coordinate, m
α = radiative absorptance
ϵ = radiative emittance
ρ = density, kg/m^3
σ = Stefan-Boltzmann constant, W/m^{2}K^4

Introduction

THE phase change coating technique has provided a valuable tool for determining the heat-transfer rate over the surface of small, complex wind-tunnel models. The phase-change technique involves coating the surface of a model, molded from a low thermal conductivity material such as stycast plastic, with a paint which changes from solid to liquid phase at a precisely known temperature.[1] When the model is subjected to aerodynamic heating in a wind tunnel, the paint melts across well-defined surface locations (isotherms) as the phase-change temperature is reached. If the time required to reach the phase-change temperature as well as the adiabatic wall temperature is known along a model surface, the aerodynamic heat-transfer coefficient may be found through an appropriate solution to the one-dimensional heat conduction equation.[2]

The phase-change paint technique has been used with reasonable success over the past several years. During that time, improvements have been made in the painting, lighting, and photographic techniques as well as in the development of new computer models to handle aerodynamic heating analyses on thin wing sections.[3,4] Even with these improvements, however, a problem still existed which limited the accuracy of the calculated heat-transfer coefficients. The problem was in directly utilizing temperature dependent values of thermal conductivity and specific heat determined by steady-state techniques. The fact that k and c as well as ρ were determined under steady-state conditions with the temperature uniform over the entire thickness of the model effectively averaged spatially the thermophysical properties, whereas large temperature gradients actually existed across the region during a wind-tunnel test. Therefore, when wind-tunnel models are exposed to aerodynamic heating, the properties are far from measured steady-state properties. In addition, the plastic models are themselves sometimes spatially nonhomogeneous because filler material is used in the resin matrix to withstand elevated temperatures in the wind tunnel. Thus, the thermophysical properties of the model material can be functions of both temperature and position, and failure to consider these dependencies can result in rather appreciable error.

The numerical technique described herein has proven that an effective thermophysical property $(k\rho c)^{1/2}$ may be determined which properly accounts for model inhomogeneity and temperature dependency in a transient environment. The analysis also confirms that the experimental apparatus described in Ref. 5 can be used to measure directly this effective thermophysical property for use in wind-tunnel model heat-transfer measurements.

Analysis

Wind-tunnel models normally are represented thermally as constant property semi-infinite slabs, implying that the heat flow into the model is one-dimensional and never fully penetrates the model centerline. The validity of these assumptions is justified by the fact that the model materials are thermally thick and have a low thermal diffusivity. Also, because of their low thermal conductivity and locally low lateral temperature gradients, the predominant direction of heat flow is in the direction of impressed surface heat flux, i.e., essentially one dimensional. Under these conditions, the heat conduction equation reduces to

$$\frac{\partial}{\partial x}\left[k(x,T)\frac{\partial T}{\partial x}(x,t)\right]=\rho(x)\,c(x,T)\frac{\partial T}{\partial t}(x,t) \tag{1}$$

Because of the nonlinearity and nonhomogeneous nature of this equation, resulting from the spatial and temperature

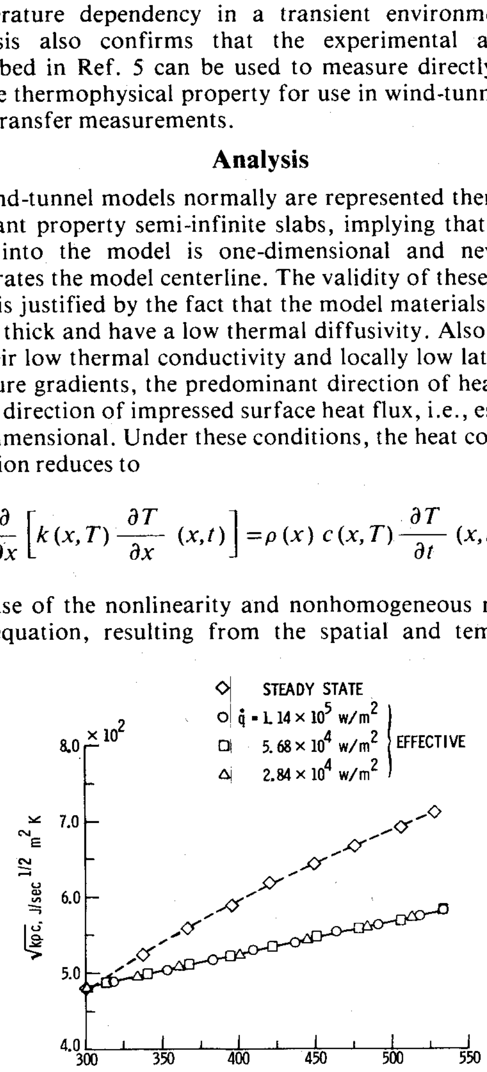

Fig. 1 Effective and measured steady-state thermophysical property for unfilled stycast sample.

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dependence of the properties, Eq. (1) was solved numerically using a finite difference scheme.[2] The slab was assumed thermally thick so that the temperature deep within the material remained constant at its initial value of T_i, or

$$\lim_{x \to \infty} T(x,t) = T_i \tag{2}$$

Either of two boundary conditions was specified for the slab surface. Under convective loading, as experienced by a model in a wind tunnel, the transient surface temperature was described by

$$-k(0,T)\frac{\partial T}{\partial x}(0,t) = h[T_{aw} - T(0,t)] - \epsilon\sigma T^4(0,t) \tag{3}$$

Under radiative surface heating conditions, Eq. (3) must be replaced by

$$-k(0,T)\frac{\partial T}{\partial x}(0,t) = \alpha\dot{q} - \epsilon\sigma T^4(0,t) \tag{4}$$

The latter surface boundary conditions simulated the experimental radiant heating apparatus which is being developed at Langley Research Center to determine the effective thermophysical property of model materials. The two boundary conditions also permitted proof of the concept that this effective thermophysical property developed from radiant heating data can be used in convective wind-tunnel calculations.

Steady-state values of thermal conductivity, specific heat, and density for the model materials under study were determined by standard techniques.[2] Thermal conductivity was measured at several temperatures with a DynaTech Comparative Thermal Conductivity Apparatus. Specific heat was found within the same temperature range using a Perkin-Elmer Differential Scanning Calorimeter. Density was determined by simply measuring the weight and volume of each sample. Thermal conductivity measurements required moderately thick sample blocks for accurate results, and therefore k could not be determined directly for thin sample sections. Spatial dependency of the properties thus was determined by removing successive sections parallel to the surface from each sample and measuring as just described, the properties of the remaining section over a range of temperatures. An electrical resistance capacitance analogy then was used to compute k, c, and ρ as a function of position for each removed section.[6] The data for k and c then were plotted against temperatures to yield temperature dependent property variations through the sample depth. Equations (1), (2), and (3) or (4) then were solved to generate transient temperature profiles under specified surface heating loads. The profiles provided the information necessary to define the effective thermophysical property. The transient surface temperature of a constant property semi-infinite slab undergoing radiative heating but having negligible surface emission is known to be[7]

$$T(0,t) = \frac{2t^{1/2}\alpha\dot{q}}{(k\rho c)^{1/2}\pi^{1/2}} + T_i \tag{5}$$

With convective heating and negligible surface emission, the surface temperature is given by[7]

$$T(0,t) = [T_{aw} - T_i]\,[1 - e^{\gamma^2}\,\mathrm{erfc}\,\gamma] + T_i \tag{6a}$$

where

$$\gamma = h(t)^{1/2}/(k\rho c)^{1/2} \tag{6b}$$

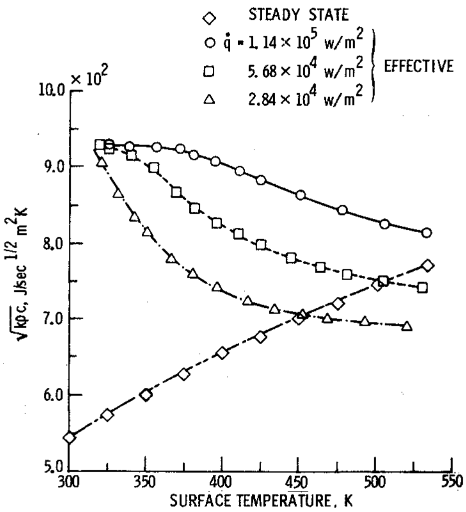

Fig. 2 Effective and measured steady-state thermophysical property for copper filled stycast sample.

Since the surface temperature histories have been calculated numerically for the actual nonhomogeneous model materials with nonlinear properties, it is possible to match the nonlinear system with the constant property system by defining an equivalent constant thermophysical property. The only unknown in either Eq. (5) or (6) is $(k\rho c)$,$^{1/2}$ and this is the required equivalent thermophysical property. Since Eq. (5) also represents the constant property solution for the experimental thermophysical property measurement device, an equivalent $(k\rho c)^{1/2}$ can be obtained from that device for comparison with the numerical solution. Subsequently, phase-change paint techniques can be used to determine the desired model surface temperature histories and, assuming the wind-tunnel adiabatic wall temperature is known, local heat-transfer coefficients can be calculated from Eq. (6) using the effective property. Results then will be identical with those obtained if the actual thermal property values had been used at each point in the slab under transient conditions. The work reported here discusses how an effective $(k\rho c)^{1/2}$ can be developed using both a numerical and experimental approach, and used in reducing phase-change heat-transfer data.

Results

The effective thermophysical property of several wind-tunnel model materials was determined in Ref. 2 for a wide range of conditions. Results of the measured steady-state variation of $(k\rho c)^{1/2}$ with temperature and the influence of surface heating rate on the effective thermophysical property are shown in Fig. 1 for a representative homogeneous model material and in Fig. 2 for an extreme nonhomogeneous model material. The assumed initial tempeaure T_i is 300° K in both cases.

Figure 1 presents results for a W-66 stycast epoxy plastic which contains no filler material and is therefore homogeneous. Examination shows the effective curve to be dependent only upon surface temperature and not on applied heat flux. Such was the case for all homogeneous model materials investigated, making tunnel data reduction quite convenient because a heat flux iteration is not required to determine $(k\rho c)^{1/2}$ as a function of surface temperature and test time. However, the significant feature of these results is the appreciable difference between the currently used steady-state measurements curve and the effective curve for increasing values of surface temperature. Recalling Eq. (6), which shows the direct dependence of the aerodynamic heat-transfer coefficient upon the effective thermophysical property, the degree of error to be expected with use of measured steady-state property values can be appreciated. These differences in $(k\rho c)^{1/2}$ appear to explain, at least in part, the discrepancies attributed to the thermal paint technique.

Figure 2 presents similar data for a W-66 stycast resin containing a copper powder filler with a large concentration gradient between the upper and lower surfaces of the sample. Whereas copper normally is not used as a filler for model fabrication, it does represent an extreme case of inhomogeneity and therefore a formidable test of the technique. Because of the material inhomogeneity, the effective curve now becomes a function of either heating rate or time as well as surface temperature. Here again, there is a significant difference between the measured steady-state thermophysical property and the effective value, particularly at the initial temperature. The use of measured steady-state properties, which are averaged over the sample thickness and are functions of overall temperature only, rather than effective properties, again would introduce significant errors into the calculation of the aerodynamic heat-transfer coefficient. In fact, nearly all model materials showed significant differences between the measured steady-state and effective properties.

Conclusions

This paper has shown that an effective thermophysical property may improve significantly the accuracy of the phase-change coating technique in estimating local heat-transfer coefficients. The apparatus described in Ref. 5 permits direct measurement of $(k\rho c)^{1/2}$ for different heating levels for models under this study. Additionally, preliminary work at Langley Research Center has shown that dynamic thermophysical property measurements can be obtained significantly faster than the steady-state techniques currently used. In light of the potential improvement in accuracy, and increased speed, the use of an effective thermophysical property to reduce phase-change paint data is recommended.

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The Response of Infinite Thin Shells to Initial Stress

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I. Introduction

SUPPOSE we have a long thin cylindrical shell which, although in the past may have been deforming plastically, is currently vibrating elastically. If this shell is deforming axisymmetrically then it is a fairly simple task to decide from an experimental strain trace what the equilibrium position would be if the experiment were continued long enough to allow the damping forces to reduce the amplitude of the vibration to insignificant values. In cases where the motion is asymmetric, however, is not quite so simple and may present a formidable analysis task if the equilibrium state is to be derived from direct observation.

The object of this paper is to compute the static equilibrium stress state of an elastically vibrating long thin cylindrical shell from the known stresses at any instant, for the general asymmetric case. Because of its previous history of plastic deformation this final equilibrium state will not be stress free. However, although the initial stress state varies with angle around the cylinder and the equilibrium strain state may therefore be quite a complex function of angle, it will be shown that the equilibrium stress must reduce to a very simple form.

II. Basic Equations

For an infinite cylindrical thin shell in a state of plane strain, the hoop mean and curvature strains are given by

$$\xi_\phi = \frac{1}{a}\frac{\partial v}{\partial \phi} - \frac{w}{a} \tag{1}$$

$$\gamma_\phi = \frac{1}{a^2}\left(\frac{\partial v}{\partial \phi} + \frac{\partial^2 w}{\partial \phi^2}\right) \tag{2}$$

where a is the cylinder radius and v and w the circumferential and radial ($+ve$ inward) displacements from the initially undeformed ring.The mean and moment of the hoop stress are given by

$$N_\phi = \frac{Et}{(1-v^2)}\xi_\phi + N'_\phi \tag{3}$$

$$M_\phi = \frac{-Et^3}{12(1-v^2)}\gamma_\phi + M'_\phi \tag{4}$$

where N_ϕ, M_ϕ is the initial stress state, E is Young's modulus, v Poisson's ratio, and t the shell thickness.

From Timoshenko[1] the full equilibrium equations for a thin cylindrical shell are

$$\frac{1}{a^2}\frac{\partial^2 v}{\partial \phi^2} - \frac{1}{a^2}\frac{\partial w}{\partial \phi} + \frac{t^2}{12a^4}\frac{\partial^3 w}{\partial \phi^3} + \frac{t^2}{12a^4}\frac{\partial^2 v}{\partial \phi^2} = -\left(\frac{1-v^2}{Et}\right)\left(\frac{1}{a}\frac{\partial N'_\phi}{\partial \phi} - \frac{1}{a^2}\frac{\partial M'_\phi}{\partial \phi}\right) \tag{5}$$

$$\frac{1}{a^2}\frac{\partial v}{\partial \phi} - \frac{w}{a^2} - \frac{t^2}{12a^4}\frac{\partial^4 w}{\partial \phi^4} - \frac{t^2}{12a^4}\frac{\partial^3 v}{\partial \phi^3} = -\left(\frac{1-v^2}{Et}\right)\left(\frac{N'_\phi}{a} + \frac{1}{a^2}\frac{\partial^2 M'_\phi}{\partial \phi^2}\right) \tag{6}$$

The initial stress state N'_ϕ, M'_ϕ may be a general function of ϕ but there is no loss of generality in assuming that each may be expanded as a Fourier series, i.e.

$$N'_\phi = N_o + \sum_{n=1,2\ldots} N_n \cos n\phi \tag{7}$$

$$M'_\phi = M_o + \sum_{n=1,2\ldots} M_n \cos n\phi \tag{8}$$

Our final equilibrium stress state may be found by superposing solutions for each of the N_i and M_i.

In the first instance let us examine the equilibrium position for an initial stress independent of ϕ, i.e. (N_0, M_0). From

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